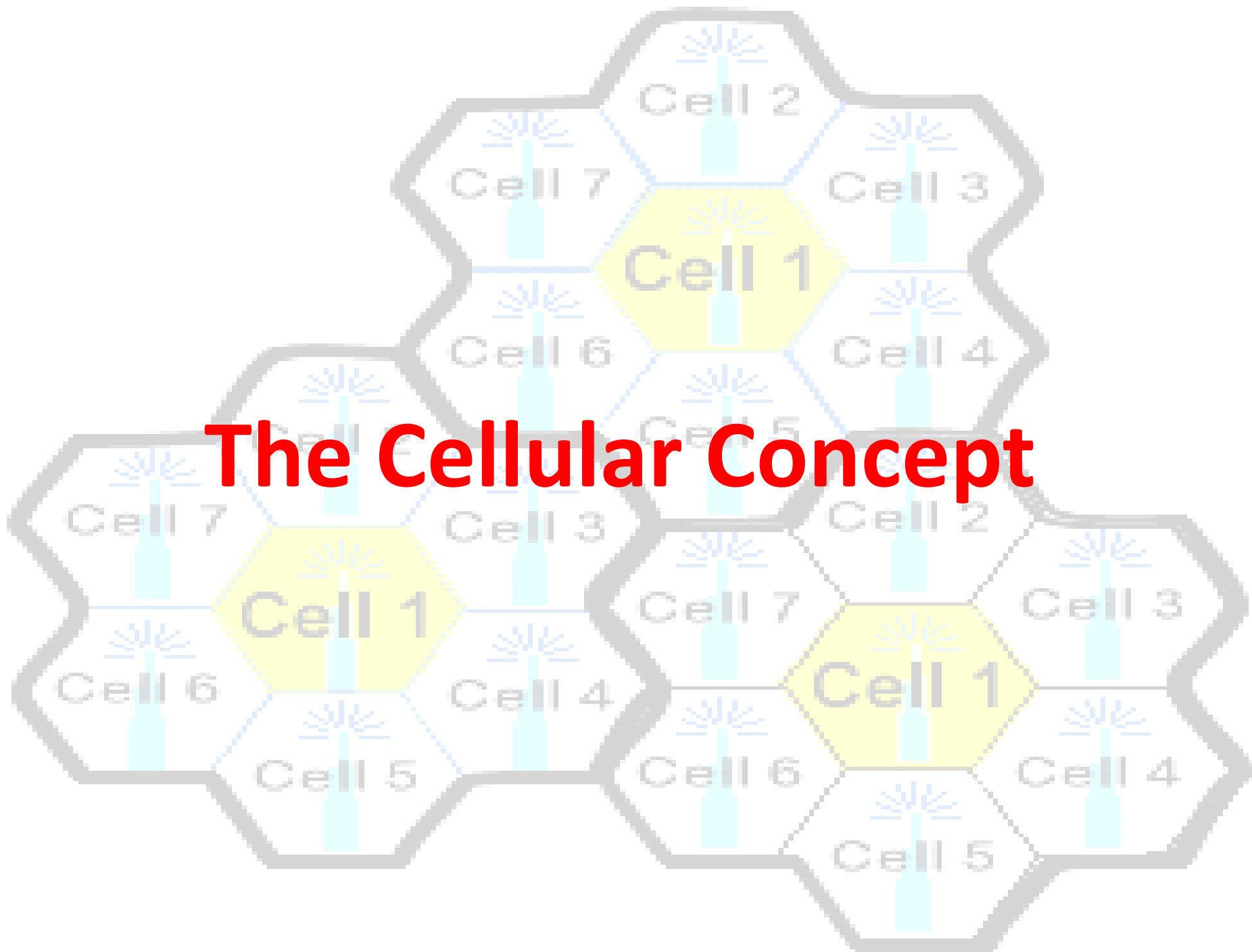


The Cellular Concept



Presentation Credits

- The material for these slides has been taken from **T. S. Rappaport's** Book: "Wireless Communications, Principles and Practice (2nd Edition)". I would like to thank the author and publishers for making the figures for this book easily accessible online.
- Content for some slides and snapshots of figures have been taken from video lectures on Wireless Communications by Dr Ranjan Bose, Dept of Electrical Engineering, IIT, Dehli

Learning Objectives

- Concept of Trunking
- Key definitions in Traffic Theory
- Erlang-(unit of traffic)
- Grade of Service
- Two Types of Trunked Systems
- Trunking Efficiency

Trunking & Grade of Service

- Cellular radio systems rely on *trunking* to accommodate a large number of users in a limited radio spectrum.
- The concept of trunking allows a large number of users to share a relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels.
- In a trunked radio system each user is allocated a channel on a per call basis, upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Trunking & Grade of Service

- The design of trunked radio systems requires understanding of trunking theory and queuing theory.
- The fundamentals of trunking theory were developed by Erlang, a Danish mathematician. The unit of traffic intensity bears his name.

Key Definitions for Trunked Radio Systems

- **Setup Time:** Time required to allocate a radio channel to a requesting user
- **Blocked Call:** A Call which cannot be completed at the time of request, due to congestion (*lost call*)
- **Holding Time:** Average duration of a typical call. Denoted by H (in seconds)
- **Request Rate:** The average number of calls requests per unit time (λ)
- **Traffic Intensity:** Measure of channel time utilization or the average channel occupancy measured in Erlangs. This is dimensionless quantity. Denoted by A
- **Load:** Traffic intensity across the entire trunked radio system (Erlangs)

Erlang-a unit of traffic

- An **Erlang** is a unit of telecommunications traffic measurement.
- Erlang represents the continuous use of one voice path.
- It is used to describe the total traffic volume of one hour
- A channel kept busy for one hour is defined as having a load of one **Erlang**
- For example, a radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic
- For 1 channel
 - Min load=0 Erlang (0% time utilization)
 - Max load=1 Erlang (100% time utilization)

Erlang-a unit of traffic

- For example, if a **group of 100 users** made 30 calls in one hour, and each call had an average call duration(holding time) of 5 minutes, then the number of Erlangs this represents is worked out as follows:
- Minutes of traffic in the hour = number of calls x duration
- Minutes of traffic in the hour = $30 \times 5 = 150$
- Hours of traffic in the hour = $150 / 60 = 2.5$
- **Traffic Intensity = 2.5 Erlangs**

Erlang-a unit of traffic

- Erlang traffic measurements are made in order to help telecommunications network designers understand traffic patterns within their voice networks.
- This is essential if they are to successfully design their network topology and establish the necessary trunk group sizes.

Traffic Concepts

- In tele-traffic we use the word traffic to denote the **traffic intensity**
- **Carried Traffic(A_c)**: Traffic carried by the group of servers during the time interval T
 - A channel can at most carry one Erlang
 - The maximum possible carried traffic is the total number of channels C , in Erlangs
 - The income is often proportional to the carried traffic
- **Offered Traffic(A)**: The traffic that would be carried if no calls were rejected due to lack of capacity. It is thus the traffic *offered* to the trunked system
 - When offered traffic exceeds the maximum capacity of system, the carried traffic becomes limited due to limited number of channels
- **Lost or Rejected Traffic**: The difference between offered and carried traffic.
 - The rejected traffic can be reduced if the capacity of system is increased

Key Definitions for Trunked Radio Systems

- **Traffic Intensity offered by each user (A_u):** Equals average call arrival rate multiplied by the holding time(service time)

$$A_u = \lambda H \quad (\text{Erlangs})$$

- **Total Offered Traffic Intensity for a system of U users (A):**

$$A = U A_u \quad (\text{Erlangs})$$

- **Traffic Intensity per channel, in a C channel trunked system**

$$A_c = U A_u / C \quad (\text{Erlangs})$$

Trunking & Grade of Service

- In a trunked radio system, when a particular user requests service and all the available radio channels are already in use , the user is *blocked* or denied access to the system. In some systems a queue may be used to hold the requesting users until a channel becomes available.
- Trunking systems must be designed carefully in order to ensure that there is a low likelihood that a user will be blocked or denied access.
- The likelihood that a call is blocked, or the likelihood that a call experiences a delay greater than a certain queuing time is called “**Grade of Service**” (GOS).

Trunking & Grade of Service

- **Grade of Service (GOS):** Measure of ability of a user to access a trunked system during the **busiest hour**. Measure of the congestion which is specified as a probability.
 - The probability of a call being blocked
 - **Blocked calls cleared or Lost Call Cleared(LCC) or Erlang B systems**
 - The probability of a call being delayed beyond a certain amount of time before being granted access
 - **Blocked call delayed or Lost Call Delayed(LCD) or Erlang C systems**

Blocked Call Cleared Systems

- Queuing is not provided for call requests
- When a user requests service, there is a minimal call set-up time and the user is given immediate access to a channel if one is available
- If channels are already in use and no new channels are available, call is blocked without access to the system
- The user does not receive service, but is free to try again later
- All blocked calls are instantly returned to the user pool
- Mathematical modelling of such systems is done by Erlang B formula

Modelling of Blocked Call Cleared Systems

The Erlang B model is based on following assumptions :

- Calls are assumed to arrive with a Poisson distribution
- There are nearly an infinite number of users
- Call requests are memory less ,implying that all users, including blocked users, may request a channel at any time
- All free channels are fully available for servicing calls until all channels are occupied
- The probability of a user occupying a channel(called service time) is exponentially distributed. Longer calls are less likely to happen
- There are a finite number of channels available in the trunking pool.

Modelling of Blocked Call Cleared Systems

The Erlang B model is based on following assumptions :

- Inter-arrival times between call requests are exponential
- Inter-arrival times of call requests are independent of each other
- The model is accurate for a large system with many channel and many users with similar calling patterns

Blocked Call Cleared Systems

- The assumptions on the previous slide lead to the Erlang B formula which determines the probability that a call is blocked and is a measure of the GOS for a trunked system which provides no queuing for blocked calls. The **Erlang B** formula is given by

$$P_r[\textit{blocking}] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^C \frac{A^k}{k!}} = \textit{GOS}$$

where C is the number of trunked channels offered by a trunked radio system and A is the total offered traffic.

Erlang B

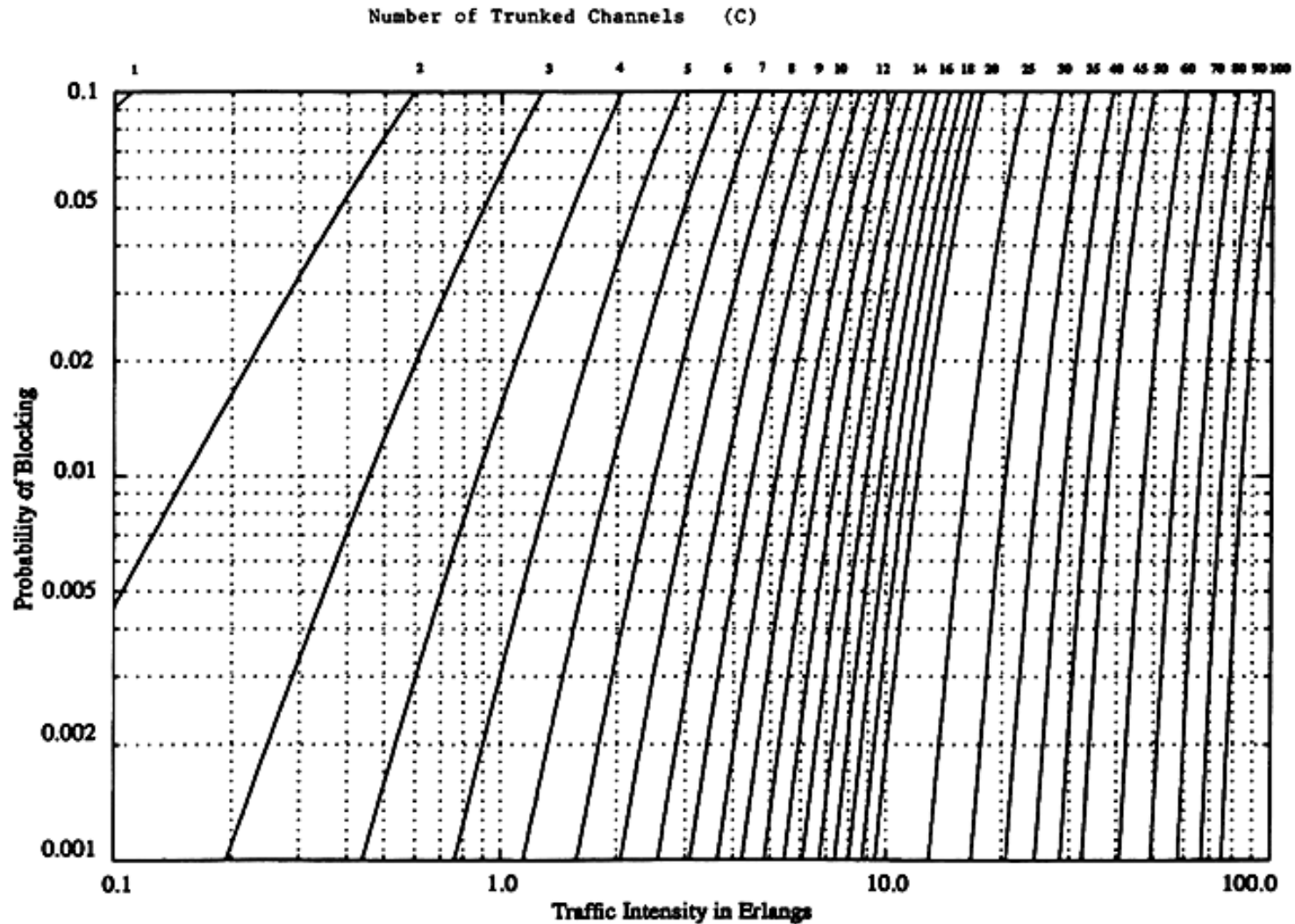


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Example 3.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 5, (b) 10. Assumed that each user generates 0.5 Erlangs of traffic.

Solution:

- Given $C=5$, $GOS=0.005$, $A_u=0.1$,
From graph using $C=5$ and $GOS=0.005$, find A

Erlang B

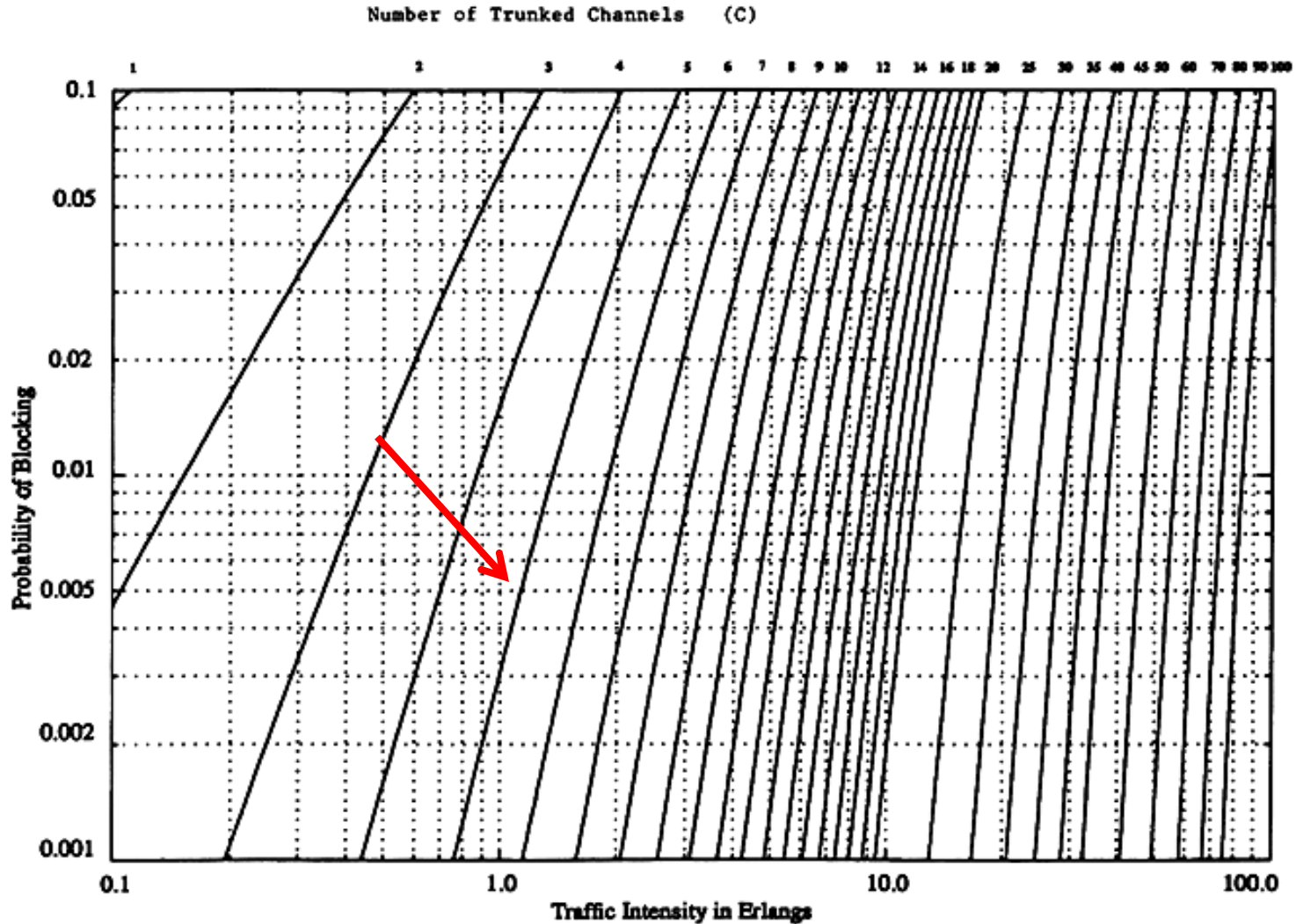


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Example 3.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 5, (b) 10. Assumed that each user generates 0.5 Erlangs of traffic.

Solution

- Given $C=5$, $GOS=0.005$, $A_u=0.1$,
From graph using $C=5$ and $GOS=0.005$, $A=1.13$
Total Number of users $U=A/A_u = 1.13/0.1=11$ users
- Given $C=10$, $GOS=0.005$, $A_u=0.1$,

Erlang B

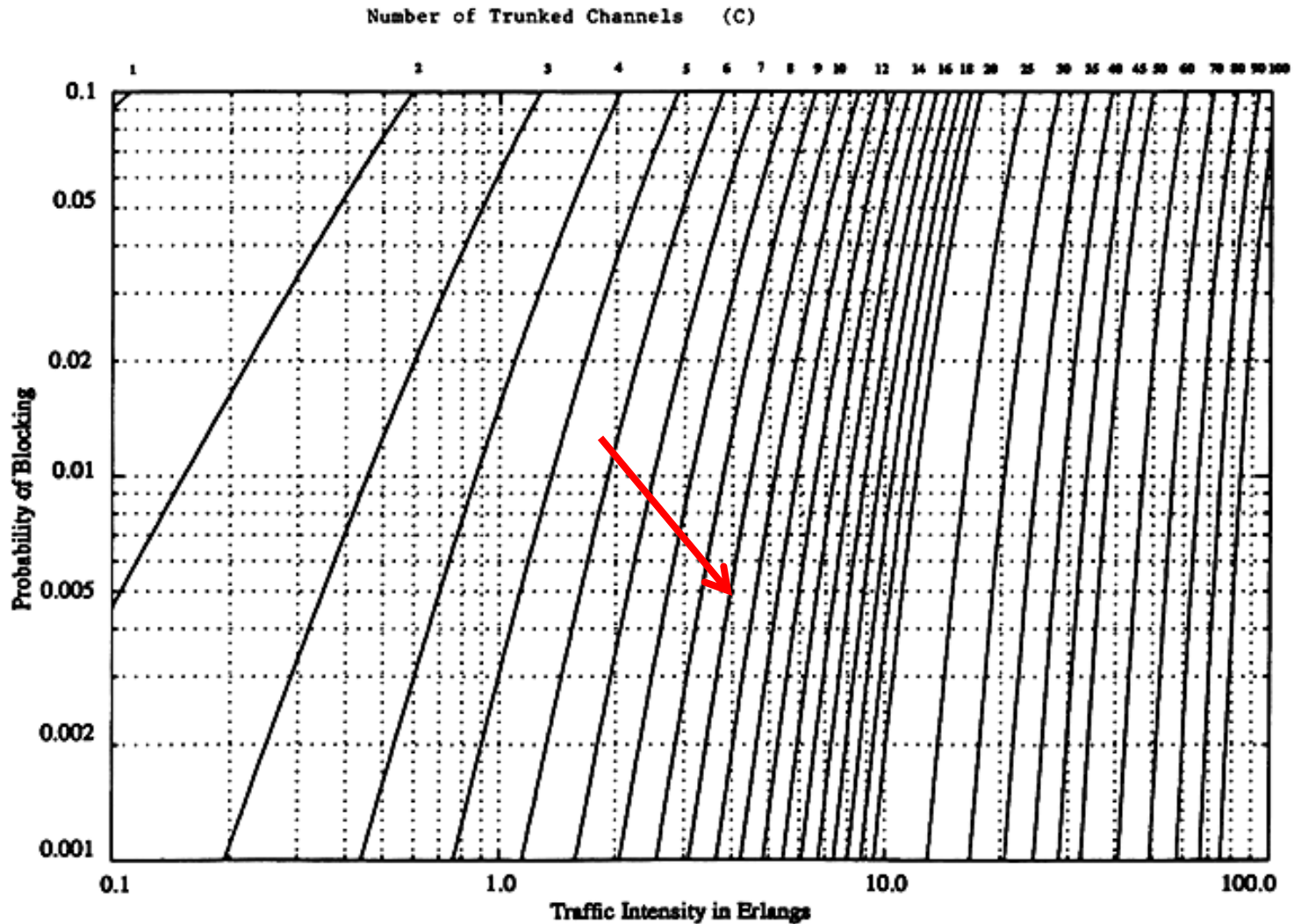


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Solution

- Given $C=5$, $GOS=0.005$, $A_u=0.1$,
From graph using $C=5$ and $GOS=0.005$, $A=1.13$
Total Number of users $U=A/A_u = 1.13/0.1=11$ users
- Given $C=10$, $GOS=0.005$, $A_u=0.1$,
From graph using $C=10$ and $GOS=0.005$, $A=3.96$
Total Number of users $U=A/A_u = 3.96/0.1=39$ users

Erlang B Trunking GOS

Table 3.4 Capacity of an Erlang B System

Number of Channels C	Capacity (Erlangs) for GOS			
	= 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

Blocked Call Delayed Systems

- Queues are used to hold call requests that are initially blocked
- When a user attempts a call and a channel is not immediately available, the call request may be delayed until a channel becomes available
- Mathematical modelling of such systems is done by Erlang C formula

Modelling of Blocked Call Delayed Systems

The Erlang C model is based on following assumptions :

- Similar to those of Erlang B
- Additionally, if offered call cannot be assigned a channel, it is placed in a queue of infinite length
- Each call is then serviced in the order of its arrival

Modelling of Blocked Call Delayed Systems

- The assumptions on the previous slide lead to the Erlang C formula which gives likelihood of a call not having immediate access to a channel (all channels are already in use)

$$P_r[\text{delay} > 0] = \frac{A^C}{A^C + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{C-1} \frac{A^k}{k!}}$$

Erlang C

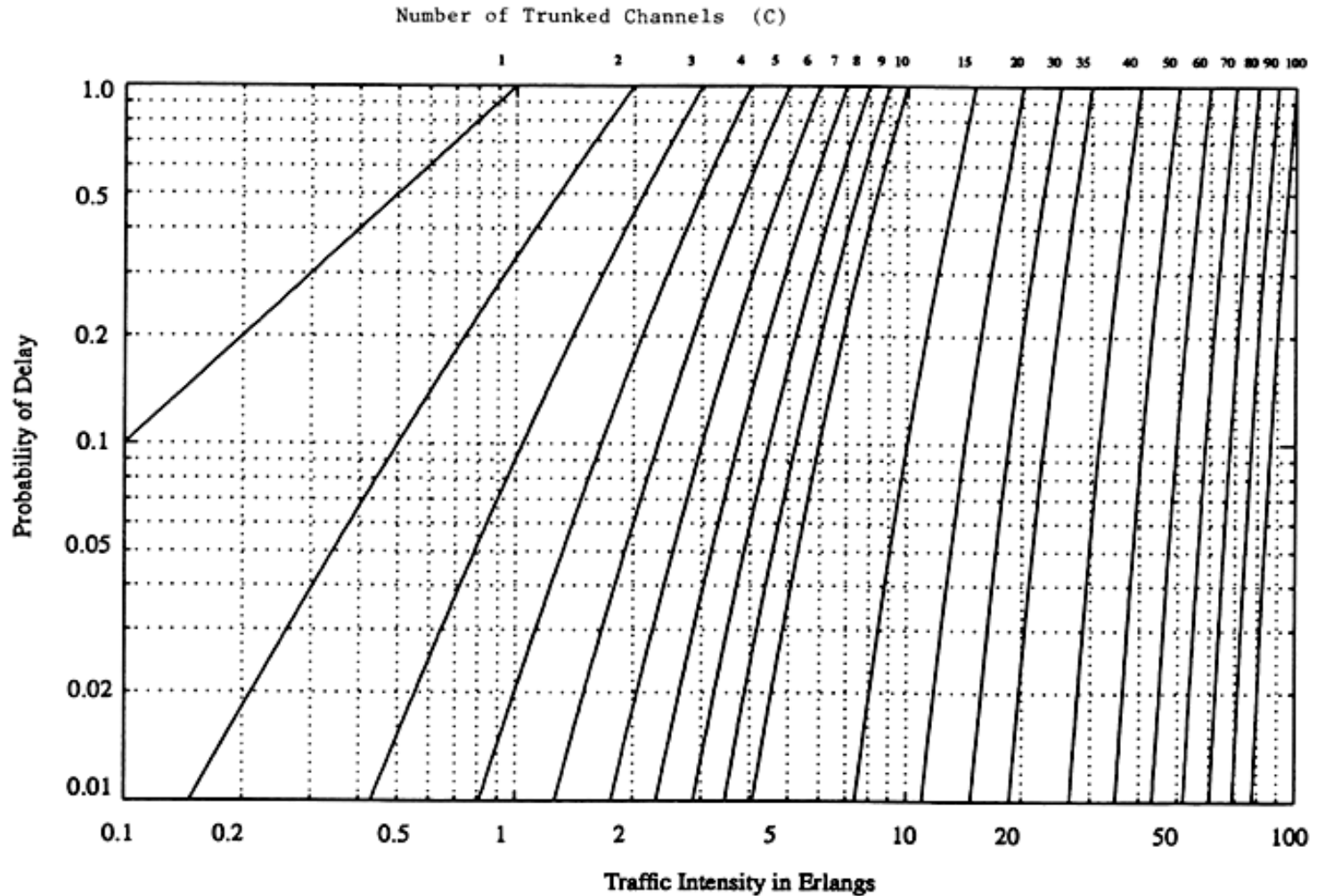


Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

Modelling of Blocked Call Delayed Systems

- Given that no channel is available initially, we have to know the probability of the call being delayed for a certain time before a channel is available for use
- So the probability that delayed call is forced to wait for more than t seconds in a queue is given by:

$$P[\text{delay} > t \mid \text{delay} > 0] = e^{-(C-A)t/H}$$

where C = total number of channels, t = delay time of interest, H = average duration of call

- Probability that any caller is delayed in queue for a wait time greater than t seconds is given as GOS of a Blocked Call Delayed System

$$P[\text{delayed call is forced to wait} > t \text{ sec}] = P[\text{delayed}] \times \text{Conditional } P[\text{delay is} > t \text{ sec}]$$

Mathematically;

$$\begin{aligned} P_r &= P_r [\text{delay} > 0] P_r [\text{delay} > t \mid \text{delay} > 0] \\ &= P_r [\text{delay} > 0] e^{-(C-A)t/H} \end{aligned}$$

Trunking Efficiency

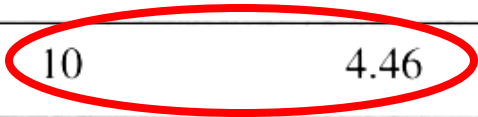
- Trunking efficiency is a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels.
- The way in which channels are grouped can substantially alter the number of users handled by a trunked system.

Erlang B Trunking GOS

Table 3.4 Capacity of an Erlang B System

Number of Channels C	Capacity (Erlangs) for GOS			
	= 0.01	= 0.005	= 0.002	= 0.001
2	2.72	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

2x1.36=2.72Erlangs



Trunking Efficiency

- **Example:**

10 trunked channels at a GOS of 0.01 can support 4.46 Erlangs of traffic, whereas two groups of 5 trunked channels can support $2 \times 1.36 = 2.72$ Erlangs of traffic

This implies that 10 trunked channels can offer 60% more traffic at a specific GOS than two 5 channel trunks.

Therefore, if in a certain situation we sub-divide the total channels in a cell into smaller channel groups then the total carried traffic will reduce with increasing number of groups



- Solve examples 3.4,3.5,3.6,3.7 on pages 80 to 86
- There shall be a quiz from Traffic Theory topic in the next class
- Bring copy of Erlang B and C charts with you
- Listen to its video explanation in Lec 5